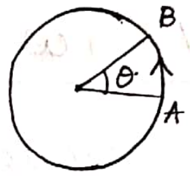
CIRCULAR MOTION

When a particle moves along a curved path, the particle is said to undergo curvilinear motion. If the curve is an arc of a circle, the motion is called circular motion. ie; Motion of a body along a circular path is known as circular motion. In this type of motion, the centre of rotation remains fixed. eg:- motion of a fan.

Angular Displacement (θ)

The displacement of a particle in circular motion is measured in terms of angular displacement θ. It is the total angle (in radians) through which the particle has rotated. Unit is radians.



Angular Velocity (ω)

The rate of change of angular displacement is termed as angular velocity. If a body is rotating with uniform angular velocity 'ω' and 'θ' is the angular displacement in 't' sec, then;

$$\boxed{\omega = \theta/t}$$

Mathematically; $\boxed{\omega = \frac{d\theta}{dt}}$

Unit is rad/sec.

Angular velocity can also be expressed in terms of revolutions per minute (rpm)

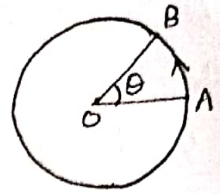
ie; $\omega = N$ rpm

$$= \frac{2\pi N}{60} \text{ rad/sec}$$

1 rev = 2π rad.

Relation between Linear velocity & angular velocity:-

Consider the body moving in a circle as shown in Fig.
The initial position of the body is at A
and after time 't', the body is at B.
The $\angle AOB = \theta$.



Angular velocity: $\omega = \theta/t$

Let V = linear velocity = $\frac{\text{linear displacement}}{\text{time}}$

But linear displacement = arc AB = $OA \times \theta$

$= \underline{r \times \theta}$ $(\because OA = \text{radius of circle} = r)$

$$\therefore V = \frac{r \times \theta}{t} \qquad (\because \theta/t = \omega)$$

$$\boxed{V = r\omega}$$

Angular Acceleration (α)

The rate of change of angular velocity is termed as angular acceleration. It may be uniform or variable.
Unit is rad/sec^2

$$\boxed{\alpha = \frac{d\omega}{dt}} = \frac{d}{dt}\left(\frac{d\theta}{dt}\right)$$

$$\boxed{\alpha = \frac{d^2\theta}{dt^2}}$$

$$\alpha = \frac{d\omega}{dt} \times \frac{d\theta}{d\theta} = \frac{d\theta}{dt} \times \frac{d\omega}{d\theta}$$

$$\therefore \boxed{\alpha = \omega \cdot \frac{d\omega}{d\theta}}$$

Relation between linear acceleration & angular acceleration:

We have; $V = r\omega$.

Differentiating w.r.t 't'

$$\frac{dV}{dt} = \frac{d}{dt}(r\omega)$$

$$= r\frac{d\omega}{dt}$$

$$\left[\because a = \frac{dV}{dt}, \quad \alpha = \frac{d\omega}{dt}\right]$$

$$\therefore \boxed{a = r\alpha}$$

$\therefore$ Linear acceleration is equal to r times angular acceleration.

Motion of rotation under constant angular acceleration (Equations of circular motion)

Consider a particle moving in a circular path.

Let ω_0 = initial angular velocity

ω = final angular velocity

t = time taken by the particle to change its velocity from ω_0 to ω.

α = angular acceleration

θ = angular displacement.

Angular acceleration = $\frac{\text{change of angular velocity}}{\text{Time}}$

$$\text{ie, } \alpha = \frac{\omega - \omega_0}{t}$$

$$\alpha t = \omega - \omega_0$$

$$\therefore \boxed{\omega = \omega_0 + \alpha t} \longrightarrow (1)$$

Angular displacement = Average angular velocity x time

$$\theta = \left(\frac{\omega_0 + \omega}{2}\right)t$$

Substituting for ω from (1)

$$\theta = \left(\frac{\omega_0 + \omega_0 + \alpha t}{2}\right)t = (\omega_0 + \tfrac{1}{2}\alpha t)t$$

$$\therefore \boxed{\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2} \longrightarrow (2)$$

Angular displacement $= \left(\frac{\omega_0 + \omega}{2}\right)t$

Substituting for t from (1)

$$\theta = \left(\frac{\omega_0 + \omega}{2}\right)\left(\frac{\omega - \omega_0}{\alpha}\right)$$

$$= \frac{\omega^2 - \omega_0^2}{2\alpha}$$

$$\therefore \boxed{\omega^2 = \omega_0^2 + 2\alpha\theta}$$

Comparison between linear motion & angular motion

SL No	Description	Linear motion	Angular motion
1.	Initial velocity	u	ω_0
2.	Final velocity	V	ω.
3.	Acceleration	a	α
4.	Displacement	s	θ.
5.	Formula for final velocity	$V = u + at$	$\omega = \omega_0 + \alpha t$
6.	Formula for distance travelled	$s = ut + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
7.	Formula for final velocity	$V^2 = u^2 + 2as$	$\omega^2 = \omega_0^2 + 2\alpha\theta$
8.	Differential formula for velocity	$V = \frac{ds}{dt}$	$\omega = \frac{d\theta}{dt}$
9.	Differential formula for acceleration	$a = \frac{dv}{dt} = V \cdot \frac{dv}{ds}$	$\alpha = \frac{d\omega}{dt} = \omega \cdot \frac{d\omega}{d\theta}$